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Calculation of pure annihilation type decay $B^+ \to D_s^+ \phi$

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Abstract. The rare decay $B^+ \to D_s^+\phi$ can occur only via annihilation-type diagrams in the standard model. We calculated this decay in the perturbative QCD approach with Sudakov resummation. We found that the branching ratio of $B^+ \to D_s^+\phi$ is of order 10^{-7}; this may be measured in the near future by the KEK and SLAC B factories. The small branching ratio predicted in the standard model makes this channel sensitive to new physics contributions.

1 Introduction

Rare B decays are useful for tests of the standard model (SM). They are sensitive to new physics contributions, since their branching ratios in SM are small. Some of them have already been measured by the CLEO and B factories in KEK and SLAC. Most of them are still under study from both the experimental and the theoretical side. Among them, the inclusive or semi-inclusive decays are clean in theory, but with more uncertainty in experimental study. On the other hand, the exclusive decays are difficult as regards precise theoretical predictions but easier as regards experimental measurement. The study of exclusive rare B decays requires knowledge of hadronization, which is non-perturbative. The generalized factorization approach has been applied to the theoretical treatment of non-leptonic B decays for some years [1]. It is a great success in explaining many decay branching ratios [2,3]. The factorization approach is a rather simple method. Some efforts have been made to improve their theoretical applications [4] and to understand the reason why the factorization approach has worked well [5,6]. One of these methods is the perturbative QCD approach (PQCD), where we can calculate the annihilation diagrams as well as the factorizable and non-factorizable diagrams.

The rare decay $B^+ \to D_s^+\phi$ is a pure annihilation-type decay. The four valence quarks in the final states D_s and ϕ are different from the ones in the B meson, i.e. there is no spectator quark in this decay. In the usual factorization approach, this decay picture is described as $\bar{b}$ and u quark in the B meson annihilating into the vacuum and the D_s and ϕ meson being produced from the vacuum afterwards. To calculate this decay in the factorization approach, one needs the $D_s \to \phi$ form factor at very large time-like momentum transfer $\mathcal{O}(M_B)$. However the form factor at such a large momentum transfer is not known in the factorization approach. This makes the factorization approach to the calculation of these decays unreliable.

In this paper, we will try to use the PQCD approach to calculate this decay. The W boson exchange causes the four quark operator transition $\bar{b}u \to \bar{s}c$; the additional $\bar{s}s$ quarks included in $D_s\phi$ are produced from a gluon. This gluon attaches to any one of the quarks participating in W boson exchange. This is shown in Fig. 1. In the rest frame of the B meson, both s and $\bar{s}$ quarks included in $D_s\phi$ have $\mathcal{O}(M_B)$ momenta, and the gluon producing them also has momentum $q^2 \sim \mathcal{O}(M_B^2)$. This is a hard gluon. One can perturbatively treat the process where the four quark operator exchanges a hard gluon with an $s\bar{s}$ quark pair. This is just the picture of the perturbative QCD approach [5, 6]. Furthermore, the final state of this decay is an isospin singlet. It is proportional to the V_{ub} transition. No V_{cb} transition can contribute to it. Therefore there will not be any dominant soft final state interaction contributions. Unlike the $B \to KK$ decays (which may have a large contribution from the final state interaction contribution) [7], the decay $B \to D_s\phi$ is a very clean channel for a test of annihilation-type contributions.

In the next section, we will show the framework of PQCD briefly. In Sect. 3, we give the analytic formulas for the decay amplitude of $B^+ \to D_s^+\phi$ decays. In Sect. 4, we give the numerical results of the branching ratio from the analytic formulas and discuss the theoretical errors. Finally, we conclude this study in Sect. 5.

2 Framework

The PQCD approach has been developed and applied in non-leptonic B meson decays [5–11] for some time. In this approach, the decay amplitude is described by three scale

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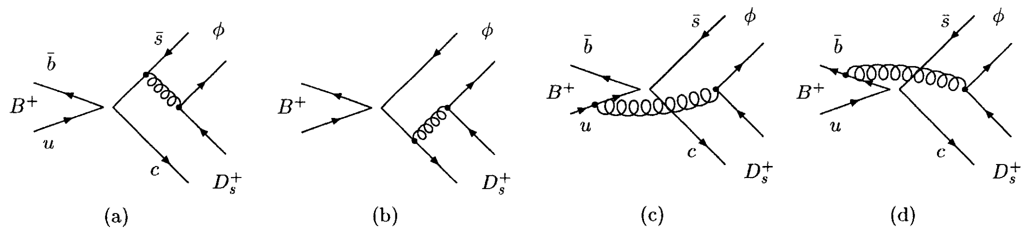

Fig. 1a–d. Diagrams for $B^+ \to D_s^+ \phi$ decay. The factorizable diagrams **a**, **b**, contribute to F_a, and the non-factorizable ones **c**, **d** to M_a

dynamics; soft (Φ), hard (H), and harder (C) dynamics. It is conceptually written as the convolution

$$\begin{aligned}\text{Amplitude} \sim & \int \mathrm{d}^4k_1 \mathrm{d}^4k_2 \mathrm{d}^4k_3 \\ & \times \operatorname{Tr}\Big[C(t)\Phi_B(k_1)\Phi_{D_s}(k_2)\Phi_\phi(k_3) \\ & \times H(k_1,k_2,k_3,t)\Big], \end{aligned} \tag{1}$$

where k_i's are the momenta of the light quarks included in each of the mesons, and Tr denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient of the four quark operator with the QCD radiative corrections. $C(t)$ includes the harder dynamics at a larger scale than M_B and describes the evolution of local 4-Fermi operators from M_W, W boson mass, down to a $t \sim \mathcal{O}(M_B)$ scale, which results in large logarithms, $\ln(M_W/t)$. H describes the four quark operator and the quark pair from the sea connected by a hard gluon whose scale is at the order of M_B, and includes the $\mathcal{O}(M_B)$ hard dynamics. Therefore, this hard part H can be perturbatively calculated. t is chosen as the largest energy scale in H, in order to lower the α_s^2 corrections to the hard part H. Φ_M is the wave function which describes hadronization of the quark and anti-quark into the meson M. While H depends on the processes considered, Φ_M is independent of the specific processes. Determining Φ_M in some other decays, we can make quantitative predictions here.

We consider the B meson at rest for simplicity. It is convenient to use the light-cone coordinates $(p^+, p^-, \boldsymbol{p}_\mathrm{T})$ to describe the meson's momenta, where $p^\pm = (p^0 \pm p^3)/(2^{1/2})$ and $\boldsymbol{p}_\mathrm{T} = (p^1, p^2)$. By these coordinates we can take the B, D_s, and ϕ mesons momenta as $P_1 = (M_B/(2^{1/2}))(1,1,\mathbf{0}_\mathrm{T})$, $P_2 = M_B/(2^{1/2})(1,r^2,\mathbf{0}_\mathrm{T})$, and $P_3 = (M_B/(2^{1/2})(0,1-r^2,\mathbf{0}_\mathrm{T})$, respectively, where $r = M_{D_s}/M_B$ and we neglect the square terms of the ϕ meson's mass M_ϕ^2. Putting the light spectator quark momenta for B, D_s and ϕ mesons k_1, k_2, and k_3, respectively, we can choose $k_1 = (0, x_1P_1^-, \boldsymbol{k}_{1\mathrm{T}})$, $k_2 = (x_2P_2^+, 0, \boldsymbol{k}_{2\mathrm{T}})$ and $k_3 = (0, x_3P_3^-, \boldsymbol{k}_{3\mathrm{T}})$. Then, integration over k_2^-, k_3^+ and k_1^+ in (1) leads to

$$\begin{aligned}\text{Amplitude} \sim & \int \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 b_3 \mathrm{d}b_3 \\ & \times \operatorname{Tr}\Big[C(t)\Phi_B(x_1,b_1)\Phi_{D_s}(x_2,b_2) \\ & \times \times \Phi_\phi(x_3,b_3)H(x_i,b_i,t)\mathrm{e}^{-S(t)}\Big], \end{aligned} \tag{2}$$

where b_i is the conjugate space coordinate of $k_{i\mathrm{T}}$. The last term, e^{-S}, contains two kinds of logarithms. One of the large logarithms is due to the renormalization of the ultraviolet divergence $\ln tb$, which describes the QCD running between scale t and $1/b$. The other is from the double logarithm due to soft gluon corrections. This double logarithm, called the Sudakov form factor, suppresses the soft dynamics effectively [12]. Thus it makes a perturbative calculation of the hard part H applicable at the intermediate scale, i.e., the M_B scale. We calculate the H for $B^+ \to D_s^+\phi$ decay in the first order in an α_s expansion and give the convoluted amplitudes in the next section.

In order to calculate analytic formulas of the decay amplitude, we use the wave functions $\Phi_{M,\alpha\beta}$ decomposed in terms of the spin structure. As a heavy meson, the B meson wave function is not well defined. It is also pointed out in the recent discussion of the B meson wave function in [13] that there is no constraint on the B meson wave function if three-parton wave functions are considered. To be consistent with previous calculations [5,6,11], we follow the same argument that the structure $(\gamma^\mu\gamma_5)_{\alpha\beta}$ and $\gamma_{5\alpha\beta}$ components make the dominant contribution in the B meson wave function. Then $\Phi_{M,\alpha\beta}$ is written

$$\Phi_{M,\alpha\beta} = \frac{\mathrm{i}}{\sqrt{2N_c}}\left\{(\not{P}_M\gamma_5)_{\alpha\beta}\phi_M^A + \gamma_{5\alpha\beta}\phi_M^P\right\}, \tag{3}$$

where $N_c = 3$ is the color degree of freedom, P_M is the corresponding meson's momentum, and $\phi_M^{A,P}$ are Lorentz scalar wave functions. As heavy quark effective theory leads to $\phi_B^P \simeq M_B\phi_B^A$, the B meson's wave function can be expressed by

$$\Phi_{B,\alpha\beta}(x,b) = \frac{\mathrm{i}}{\sqrt{2N_c}}\left[\not{P}_1 + M_B\right]\gamma_{5\alpha\beta}\phi_B(x,b). \tag{4}$$

According to [14], a pseudo-scalar meson moving fast is parameterized by Lorentz scalar wave functions, ϕ, ϕ_p, and ϕ_σ as follows:

$$\begin{aligned}&\langle D_s^-(P)|\bar{s}(z)\gamma_\mu\gamma_5 c(0)|0\rangle \\ &\quad \simeq -\mathrm{i}f_{D_s}P_\mu\int_0^1 \mathrm{d}x \mathrm{e}^{\mathrm{i}xPz}\phi(x), \end{aligned} \tag{5}$$

$$\begin{aligned}&\langle D_s^-(P)|\bar{s}(z)\gamma_5 c(0)|0\rangle \\ &\quad = -\mathrm{i}f_{D_s}m_{0D_s}\int_0^1 \mathrm{d}x \mathrm{e}^{\mathrm{i}xPz}\phi_p(x), \end{aligned} \tag{6}$$

$$\langle D_s^-(P)|\bar{s}(z)\gamma_5\sigma_{\mu\nu}c(0)|0\rangle$$

$$= \frac{\mathrm{i}}{6} f_{D_s} m_{0D_s} \left(1 - \frac{M_{D_s}^2}{m_{0D_s}^2}\right) (P_\mu z_\nu - P_\nu z_\mu) \times \int_0^1 \mathrm{d}x \mathrm{e}^{\mathrm{i}xPz} \phi_\sigma(x), \tag{7}$$

where $m_{0D_s} = M_{D_s}^2/(m_c + m_s)$. We ignore the difference between the c quark's mass and D_s meson's mass in the perturbative calculation. This means $M_{D_s} = m_{0D_s}$. In this approximation, the contributions of (7) are negligible. With the equation of motion (5) and (6), we are lead to

$$\phi_p(x) = \phi(x) + \mathcal{O}\left(\frac{\bar{\Lambda}}{M_{D_s}}\right). \tag{8}$$

Therefore the D_s meson's wave function can be expressed by one Lorentz scalar wave function,

$$\Phi_{D_s,\alpha\beta}(x,b) = \frac{\mathrm{i}}{\sqrt{2N_c}} \left[(\gamma_5 \not{P}_2)_{\alpha\beta} + M_{D_s}\gamma_{5\alpha\beta}\right] \Phi_{D_s}(x,b). \tag{9}$$

The wave function Φ_M for the $M = B, D_s$ meson is normalized by its decay constant f_M:

$$\int_0^1 \mathrm{d}x \Phi_M(x, b=0) = \frac{f_M}{2\sqrt{2N_c}}. \tag{10}$$

In contrast to the B and D_s meson, for the ϕ meson, being light, the $\sigma_{\alpha\beta}^{\mu\nu}$ component remains. In $B^+ \to D_s^+\phi$ decay, the ϕ meson is longitudinally polarized. Then, the ϕ meson's wave function is parameterized by three Lorentz structures:

$$\frac{M_\phi \not{\epsilon}}{\sqrt{2N_c}} \Phi_\phi(x_3), \quad \frac{\not{\epsilon} \not{P}_3}{\sqrt{2N_c}} \Phi_\phi^t(x_3), \quad \frac{M_\phi}{\sqrt{2N_c}} \Phi_\phi^s(x_3). \tag{11}$$

In the numerical analysis we will use Φ_ϕ, Φ_ϕ^t and Φ_ϕ^s which were calculated from QCD sum rules [16]. They will be shown in Sect. 4.

3 Perturbative calculations

The effective Hamiltonian related to $B^+ \to D_s^+\phi$ decay is given by [15]

$$H_{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} V_{ub}^* V_{cs} \left[C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)\right], \tag{12}$$

$$O_1 = (\bar{b}\gamma_\mu P_{\mathrm{L}} s)(\bar{c}\gamma^\mu P_{\mathrm{L}} u), \quad O_2 = (\bar{b}\gamma_\mu P_{\mathrm{L}} u)(\bar{c}\gamma^\mu P_{\mathrm{L}} s), \tag{13}$$

where $C_{1,2}(\mu)$ are Wilson coefficients at the renormalization scale μ. The projection operator is defined by $P_{\mathrm{L}} = 1 - \gamma_5$. The lowest order diagrams contributing to $B^+ \to D_s^+\phi$ are drawn in Fig.1 according to this effective Hamiltonian. As stated above, $B^+ \to D_s^+\phi$ decay only has annihilation diagrams.

We get the following analytic formulas by calculating the hard part H at first order in α_{s}. Together with the meson wave functions, the amplitude for the factorizable annihilation diagram in Fig.1a and b leads to

$$\begin{aligned} F_a = {} & 16\pi C_F f_B M_B^2 \int_0^1 \mathrm{d}x_2 \mathrm{d}x_3 \int_0^\infty b_2 \mathrm{d}b_2 b_3 \mathrm{d}b_3 \Phi_{D_s}(x_2, b_2) \\ & \times \Big[\Big\{x_3 \Phi_\phi(x_3, b_3) + r\,(2x_3 - 1)\, r_\phi \Phi_\phi^t(x_3, b_3) \\ & + r(1 + 2x_3) r_\phi \Phi_\phi^s(x_3, b_3)\Big\} E_f(t_a^1) h_a(x_2, x_3, b_2, b_3) \\ & - \Big\{x_2 \Phi_\phi(x_3, b_3) + 2r(1 + x_2) r_\phi \Phi_\phi^s(x_3, b_3)\Big\} \\ & \times E_f(t_a^2) h_a(x_3, x_2, b_3, b_2)\Big], \end{aligned} \tag{14}$$

where $C_F = 4/3$ is the group factor of the $\mathrm{SU}(3)_c$ gauge group, and $r_\phi = m_\phi/M_B$. The functions E_f, $t_a^{1,2}$, h_a are given in the appendix. Since we only include twist 2 and twist 3 contributions in our PQCD approach, all the r^2 and r_ϕ^2 terms in the calculation are neglected for consistence. The explicit form for the wave functions, Φ_M, is given in the next section. From (14), one can see that the factorizable contribution F_a is independent of the B meson wave function, but proportional to the B meson decay constant f_B.

The amplitude for the non-factorizable annihilation diagram in Fig.1c and d is given by

$$\begin{aligned} M_a = {} & \frac{1}{\sqrt{2N_c}} 64\pi C_F M_B^2 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \\ & \times \int_0^\infty b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \phi_B(x_1, b_1) \Phi_{D_s}(x_2, b_2) \\ & \times \Big[\Big\{x_2 \Phi_\phi(x_3, b_2) + r\,(x_2 - x_3)\, r_\phi \Phi_\phi^t(x_3, b_2) \\ & + r\,(x_2 + x_3)\, r_\phi \Phi_\phi^s(x_3, b_2)\Big\} \\ & \times E_m(t_m^1) h_a^{(1)}(x_1, x_2, x_3, b_1, b_2) \\ & - \Big\{x_3 \Phi_\phi(x_3, b_2) - r\,(x_2 - x_3)\, r_\phi \Phi_\phi^t(x_3, b_2) \\ & + r\,(2 + x_2 + x_3)\, r_\phi \Phi_\phi^s(x_3, b_2)\Big\} \\ & \times E_m(t_m^2) h_a^{(2)}(x_1, x_2, x_3, b_1, b_2)\Big]. \end{aligned} \tag{15}$$

Unlike the factorizable contribution F_a, the non-factorizable annihilation diagrams involve all three meson wave functions.

Thus, the total decay amplitude A and decay width Γ for $B^+ \to D_s^+\phi$ decay are given by

$$A = F_a + M_a, \tag{16}$$

$$\Gamma(B^+ \to D_s^+\phi) = \frac{G_{\mathrm{F}}^2 M_B^3}{128\pi} |V_{ub}^* V_{cs} A|^2, \tag{17}$$

where the overall factor is included in the decay width with the kinematics factor.

The decay amplitude for the CP conjugated mode, $B^- \to D_s^-\phi$, is the same expression as $B^+ \to D_s^+\phi$, just replacing $V_{ub}^* V_{cs}$ with $V_{ub} V_{cs}^*$. Since there is only one kind of CKM phase involved in the decay, there is no CP violation in the standard model for this channel. We thus have $\mathrm{Br}(B^+ \to D_s^+\phi) = \mathrm{Br}(B^- \to D_s^-\phi)$.

4 Numerical results

In this section we show numerical results obtained from the previous formulas. To begin, we give the branching ratios predicted from the same parameters and wave functions as are adopted in other works. Secondly, we discuss the theoretical errors due to uncertainty of some parameters.

For the B meson's wave function, there is a sharp peak at the small x region; we use

$$\Phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \quad (18)$$

which is adopted in [5,6,11]. This choice of the B meson's wave function is almost a best fit from the $B \to K\pi$, $\pi\pi$, $\pi\rho$ and $\pi\omega$ decays. For the D_s meson's wave function, we assume the form to be as follows, leaving a_{D_s} as a free parameter:

$$\Phi_{D_s}(x,b) = \frac{3}{\sqrt{2N_c}} f_{D_s} x(1-x)\{1 + a_{D_s}(1-2x)\} \times \exp\left[-\frac{1}{2}(\omega_D b)^2\right]. \quad (19)$$

This is a rather flat distribution function. Since the c quark is heavier than the s quark, this function is peaked at the c quark side, i.e. the small x region. The wave functions of the ϕ meson are derived by the QCD sum rules [16]:

$$\Phi_\phi(x) = \frac{f_\phi}{2\sqrt{2N_c}} 6x(1-x), \quad (20)$$

$$\Phi_\phi^t(x) = \frac{f_\phi^T}{2\sqrt{2N_c}} \left\{3\xi^2 + 0.21\left(3 - 30\xi^2 + 35\xi^4\right) + 0.69\left(1 + \xi \ln\frac{x}{1-x}\right)\right\}, \quad (21)$$

$$\Phi_\phi^s(x) = \frac{f_\phi^T}{4\sqrt{2N_c}} \left\{3\xi\left(4.5 - 11.2x + 11.2x^2\right) + 1.38 \ln\frac{x}{1-x}\right\}, \quad (22)$$

where $\xi = 1 - 2x$. In addition, we use the following input parameters:

$$M_B = 5.279\,\text{GeV}, \quad M_{D_s} = 1.969\,\text{GeV}, \quad m_\phi = 1.02\,\text{GeV}, \quad (23)$$

$$f_B = 190\text{MeV}, \quad f_\phi = 237\text{MeV}, \quad f_\phi^T = 220\text{MeV}, \quad f_{D_s} = 241\,\text{MeV}, \quad (24)$$

$$\omega_b = 0.4\,\text{GeV}, \quad a_{D_s} = 0.3, \quad \omega_D = 0.2\,\text{GeV}. \quad (25)$$

With these values and (10) we get the normalization factor $N_B = 91.745\,\text{GeV}$. Using the above fixed parameters, we find that the factorizable annihilation diagram contribution is dominant over the non-factorizable contribution. The reason is that the Wilson coefficient in the non-factorizable contribution M_a is $C_1(t)$, which is smaller than the one in the factorizable contribution F_a, $a_1 = C_1/3 + C_2$. Although the real part of M_a is negligible, the imaginary part of M_a is comparable with the imaginary part of F_a; it is about 30% of the real part of F_a.

The propagators of the inner quark and gluon in Fig. 1 are usually proportional to $1/x_i$. One may suspect that these amplitudes are enhanced by the endpoint singularity around $x_i \sim 0$. This can be explicitly found in (A.8) and (A.9), where the Bessel function Y_0 diverges at $x_i \sim 0$ or 1. However this is so in our calculation. First we introduce the transverse momentum of the quark, such that the propagators become $1/(x_i x_j + k_\mathrm{T}^2)$. There is no divergence at the endpoint region. Secondly, the Sudakov form factor $\exp[-S]$ suppresses the region of small k_T^2. Therefore there is no singularity in our calculation. We also include the threshold resummation in our calculation of factorizable diagrams, which further suppresses the endpoint region contribution [17]. The dominant contribution is not from the endpoint region of the wave function. As a proof, in our numerical calculations, for example, an expectation value of α_s in the integration for F_a and M_a results in $\langle\alpha_\mathrm{s}/\pi\rangle \simeq 0.1$. Therefore, the perturbative calculations are self-consistent.

Now we can calculate the branching ratio according to (16) and (17). Here we use the CKM matrix elements [18]

$$|V_{ub}| = 0.0036 \pm 0.0010, \quad |V_{cs}| = 0.9891 \pm 0.016, \quad (26)$$

and the life time for the $B^\pm$ meson is $\tau_{B^\pm} = 1.65 \times 10^{-12}$ s. The predicted branching ratio is

$$\mathrm{Br}(B^+ \to D_s^+ \phi) = 3.0 \times 10^{-7}. \quad (27)$$

This is still far from the current experimental upper limit [18]

$$\mathrm{Br}(B^+ \to D_s^+ \phi) < 3.2 \times 10^{-4}. \quad (28)$$

The branching ratios obtained from the analytic formulas may be sensitive to various parameters, such as the parameters in (25). Uncertainty of the predictions on PQCD is mainly due to the meson wave functions. Therefore it is important to give the limits of the branching ratio when we choose the parameters to the appropriate extent. The appropriate extent of ω_b can be obtained from calculation of the semi-leptonic decays [19] and the other $B \to \pi\pi$, $B \to K\pi$ and $B \to \rho\pi$, $\omega\pi$ decays [5,6,11]:

$$0.35\,\text{GeV} \le \omega_b \le 0.45\,\text{GeV}. \quad (29)$$

The change of value of ω_b will not alter the result of F_a, which is independent of the B meson wave function, but will affect the value of M_a. We did not find any strict constraints for the D_s meson wave function in the literature. In fact, a future study of $B \to D_s\pi$ will do this job. At present, a_{D_s} in the D_s meson wave function is a free parameter, and we take $0 \le a_{D_s} \le 1$. Here we check the sensitivity of our predictions on ω_b and a_{D_s} within the ranges stated above. The branching ratios normalized by the decay constants and the CKM matrix elements can result in

$$\mathrm{Br}(B^+ \to D_s^+\phi) = (3.0^{+2.4}_{-1.0}) \times 10^{-7} \times \left(\frac{f_B \quad f_{D_s}}{190\,\mathrm{MeV} \cdot 241\,\mathrm{MeV}}\right)^2 \left(\frac{|V_{ub}^* \quad V_{cs}|}{0.0036 \cdot 0.9891}\right)^2. \tag{30}$$

Considering the uncertainty of f_B, f_{D_s} and $|V_{ub}^* V_{cs}|$ et cetera, the branching ratio of the $B^+ \to D_s^+\phi$ decay is at the order of 10^{-7}. This may be measured by the current B factory experiments in KEK and SLAC.

5 Conclusion

In two-body hadronic B meson decays, the final state mesons are moving very fast, since each of them carries more than 2 GeV energy. There is not enough time for them to exchange soft gluons. The soft final state interaction is not important in the two-body B decays. This is consistent with the argument based on color transparency [20]. The PQCD with Sudakov form factor is a self-consistent approach to the description of the two-body B meson decays. Although the annihilation diagrams are suppressed comparing to other spectator diagrams, their contributions are not negligible in the PQCD approach [5, 6].

In this paper, we calculate the $B^+ \to D_s^+\phi$ decay in the PQCD approach. Since neither of the bottom quark or the up quark in the initial B meson appeared in the final mesons, this process occurs purely via annihilation type diagrams. It is a charm quark (not an anti-charm quark) in the final states, therefore the usual V_{cb} transition does not contribute to this process. The final states are isospin singlet. There should be no dominant final state interactions through which other channels contribute. From our PQCD study, the branching ratio of $B^+ \to D_s^+\phi$ decay is still sizable with a branching ratio around 10^{-7}, which may be measured in the current running B factories Belle, BABAR or in LHC-B in the future. This may be one of the channels to be measured in B decays via the annihilation-type diagram. Whether the PQCD predicted branching ratio is good enough to account for the $B^+ \to D_s^+\phi$ decay will soon be tested in the current or future experiments.

The small branching ratio (comparing to the already measured other B decays) predicted in the SM, makes this channel sensitive to any new physics contributions. Since the CP asymmetry predicted for this channel in the SM is zero, any non-zero measurement of CP asymmetry will be a definite signal of new physics. We also notice that the supersymmetric contribution will not enhance the decay branching ratio significantly, but it may contribute to a non-zero CP asymmetry in this channel, since the supersymmetry couplings can introduce new phases.

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Appendix A: Some functions

The definitions of some functions used in the text are presented in this appendix. In the numerical analysis we use the one-loop expression for the strong coupling constant:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)}, \tag{A.1}$$

where $\beta_0 = (33 - 2n_f)/3$ and n_f is number of active quark flavors at the appropriate scale. Λ is the QCD scale, which we take as 250 MeV at $n_f = 4$. We also use the leading logarithms expressions for the Wilson coefficients $C_{1,2}$ presented in [15]. Then we put $m_t = 170\,\mathrm{GeV}$, $m_W = 80.2\,\mathrm{GeV}$, $m_b = 4.8\,\mathrm{GeV}$, and $m_c = 1.3\,\mathrm{GeV}$ in the Wilson coefficients calculation.

The function E_f and E_m are defined by

$$E_f(t) = [C_1(t)/3 + C_2(t)]\, \alpha_s(t) \mathrm{e}^{-S_D(t)-S_\phi(t)}, \tag{A.2}$$

$$E_m(t) = C_1(t)\alpha_s(t)\mathrm{e}^{-S_B(t)-S_D(t)-S_\phi(t)}. \tag{A.3}$$

The above $S_{B,D,\phi}$ are defined by

$$S_B(t) = s(x_1 P_1^+, b_1) + 2\int_{1/b_1}^{t} \frac{\mathrm{d}\mu'}{\mu'} \gamma_q(\mu'), \tag{A.4}$$

$$S_D(t) = s(x_2 P_2^+, b_3) + 2\int_{1/b_2}^{t} \frac{\mathrm{d}\mu'}{\mu'} \gamma_q(\mu'), \tag{A.5}$$

$$S_\phi(t) = s(x_3 P_3^+, b_3) + s((1-x_3)P_3^+, b_3) + 2\int_{1/b_3}^{t} \frac{\mathrm{d}\mu'}{\mu'} \gamma_q(\mu'), \tag{A.6}$$

where the last terms of the above formulas are logarithms from the renormalization of ultraviolet divergence. The term $s(Q,b)$, the so-called Sudakov factor, results from summing up the double logarithms caused by a collinear divergence and a soft divergence. The expression is given by [10]

$$s(Q,b) = \int_{1/b}^{Q} \frac{\mathrm{d}\mu'}{\mu'} \left[\left\{ \frac{2}{3}(2\gamma_\mathrm{E} - 1 - \log 2) + C_F \log \frac{Q}{\mu'} \right\} \times \frac{\alpha_s(\mu')}{\pi} + \left\{ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3}\beta_0 \log \frac{\gamma_\mathrm{E}}{2} \right\} \times \left(\frac{\alpha_s(\mu')}{\pi}\right)^2 \log \frac{Q}{\mu'} \right]; \tag{A.7}$$

$\gamma_\mathrm{E} = 0.57722\cdots$ is the Euler constant, and $\gamma_q = \alpha_s/\pi$ is the quark anomalous dimension.

The h's in the decay amplitudes are given by performing a Fourier transformation on the transverse momenta

$\boldsymbol{k}_{i\mathrm{T}}$ for the propagators of the virtual quark and gluon in the hard part calculation; they result in

$$\begin{aligned} & h_a(x_2, x_3, b_2, b_3) \\ & = (\pi \mathrm{i}/2)^2 \, H_0^1(M_B\sqrt{x_2 x_3} b_2) S_t(x_3) \\ & \quad \times \Big\{ H_0^1(M_B\sqrt{x_3} b_2) J_0(M_B\sqrt{x_3} b_3)\theta(b_2 - b_3) \\ & \quad + (b_2 \leftrightarrow b_3) \Big\}, \end{aligned} \tag{A.8}$$

$$\begin{aligned} & h_a^{(j)}(x_1, x_2, x_3, b_1, b_2) \\ & = \begin{pmatrix} \mathrm{K}_0(M_B\sqrt{F_j} b_1), & \text{for} \quad F_j \geq 0, \\ \frac{\pi \mathrm{i}}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{-F_j} b_1), & \text{for} \quad F_j < 0, \end{pmatrix} \\ & \quad \times \Big\{ \frac{\pi \mathrm{i}}{2} \mathrm{H}_0^{(1)}(M_B\sqrt{x_2 x_3} b_1) \mathrm{J}_0(M_B\sqrt{x_2 x_3} b_2)\theta(b_1 - b_2) \\ & \quad + (b_1 \leftrightarrow b_2) \Big\}, \end{aligned} \tag{A.9}$$

with the variables $F_1 = x_2(x_1 - x_3)$, $F_2 = x_2 + (1 - x_2)(x_1 + x_3)$, and $\mathrm{H}_0^{(1)}(z) = \mathrm{J}_0(z) + \mathrm{i}\mathrm{Y}_0(z)$. The threshold resummation form factor $S_t(x_i)$ is adopted from [19]:

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)}[x(1 - x)]^c, \tag{A.10}$$

where the parameter $c = 0.3$. This function is normalized to unity. The hard scales t in the amplitudes are taken as the largest energy scale in H to diminish the higher order α_s^2 corrections:

$$t_a^1 = \max(M_B\sqrt{x_3}, 1/b_2, 1/b_3), \tag{A.11}$$
$$t_a^2 = \max(M_B\sqrt{x_2}, 1/b_2, 1/b_3), \tag{A.12}$$
$$t_m^1 = \max(M_B\sqrt{|F_1|}, M_B\sqrt{x_2 x_3}, 1/b_1, 1/b_2), \tag{A.13}$$
$$t_m^2 = \max(M_B\sqrt{F_2}, M_B\sqrt{x_2 x_3}, 1/b_1, 1/b_2). \tag{A.14}$$

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